DIFFERENTIAL TOPOLOGY MID-TERM EXAM

M.MATH II - RAMESH SREEKANTAN

Each question is of 8 marks adding up to a total of 40 marks. Feel free to *use* theorems you have been taught. However, if you do apply a theorem you must state the theorem precisely and show that it applies to the situation at hand. And please do not cheat.

1. Let $\mathbb{S}^1$ be the unit circle in $\mathbb{R}^2$. Prove that $\mathbb{S}^1 \times \mathbb{S}^1$ is a manifold. (8)

2. If X is compact and Y is connected, show that every *submersion*

$$f : X \longrightarrow Y$$

is surjective. (8)

3a. Let

$$f : \mathbb{S}^1 \longrightarrow \mathbb{S}^1$$
$$f(x) = -x$$

be the *antipodal map*. Show that it is homotopic to the identity. (4)

3b. Show that if k is *odd* the same is true for the antipodal map $\mathbb{S}^k \longrightarrow \mathbb{S}^k$. (4)

4. Show that $[0, 1] \times [01,]$ is *not* a manifold with boundary. (8)

5. Show that the Brouwer Fixed Point Theorem is *false* for the *open* ball of radius $a > 0$ (8)

$$\mathbb{B}^k = \{x \in \mathbb{R}^k | \|x\| < a\}$$

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